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## Research Report

# Arithmetic procedural knowledge: A cortico-subcortical circuit

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### ABSTRACT

The disturbances of arithmetic procedural knowledge form a heterogeneous picture, in which we can distinguish "memory" impairments and "monitoring" problems. Patients with "memory" disturbances reported in the literature present left parietal lesions, while "monitoring" impairments have been assumed to be due to frontal damage. Procedural knowledge has been less investigated in basal ganglia lesions, in which there has been no analysis of procedural impairments. The present study investigates and compares the patterns of acalculia in two patients, one with a left parietal lesion and the other with a left basal ganglia lesion. The patients were tested on a broad range of neuropsychological abilities, with the main focus on number processing and calculation. The results show many similarities between their deficits, with some difficulties in simple arithmetic, arithmetical rules and mental and written complex calculations. The errors made in complex mental and written calculations were due to memory-based procedural impairments in both patients. These findings, corroborated with other studies reported in the literature, suggest the existence of a fronto-parieto-subcortical circuit responsible for arithmetic complex calculations and that procedural knowledge relies on a visuo-spatial sketchpad that contains a representation of each sub-step of the procedure.

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## 1. Introduction

In recent years, the study of number processing and calculation skills has made considerable progress. It is now well established that the ability to solve a complex arithmetical operation is a multi-componential process that includes different types of knowledge: arithmetical facts (e.g. multiplication tables), procedural knowledge (e.g. the use of carrying and borrowing) and the understanding of arithmetic operations and the laws concerning these operations, a process known as "conceptual" knowledge (Sokol and McCloskey, 1991). The existence of relative independence between these three components has been supported by many number processing and calculation studies in brain-damaged patients.

Arithmetic procedural knowledge consists of the algorithms required to solve multidigit calculations. It is referred to as a routine expertise (Hatano, 1988) because it does not require conceptual understanding of each step of the procedure and is applied in familiar contexts.

Many papers have reported the dissociation between procedural knowledge and fact retrieval with impaired procedures in cortical lesions (McCloskey et al., 1985; Caramaizza and McCloskey, 1987; Grafman et al., 1989; Lucchelli and DeRenzi, 1993; Cipolotti et al., 1994; Girelli and Delazer, 1996; Semenza et al., 1997; McNeil and Burgess, 2002; Sandrini et al., 2003; Granà et al., 2006; Delazer et al., 2006; Roșca, 2009).

The disturbances of arithmetic procedural knowledge form a heterogeneous picture. "Memory" impairments in

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procedural algorithms can be distinguished from “monitoring” problems (Semenza et al., 1997). Patients with “memory” impairments (Girelli and Delazer, 1996; Sandrini et al., 2003) commit consistent and systematic errors (“bugs”) generated by the systematic misapplication of a specific rule. Some subprocedures are always impaired in the same way and others are correct. Their performance does not decrease as the operation proceeds and they know when an operation or sub-step of an operation is completed. They are usually aware of their difficulties. In contrast, “monitoring” patients (Semenza et al., 1997) make inconsistent and unsystematic errors, all subprocedures being occasionally correct. They begin the operation correctly but make an increasing number of errors of all kind. They have difficulties ending the operation and have little awareness about the precision of their performance. “Memory” problems in procedural knowledge have been reported in patients with left parietal and left temporo-parietal lesions, while “monitoring” impairments have been assumed to be due to frontal lesions. Recently, it was suggested that lesions of the right parietal lobe might compromise the spatial schema that is used in overcoming working memory demands of complex calculations by representing the information of “where” exactly each sub-step should be placed (Granà et al., 2006). Furthermore, other authors reporting a case of a patient with a left parietal lesion and severely impaired procedural knowledge, hypothesized that the left parietal lobe is involved in “how, what and when” to compute while the right parietal lobe supports the knowledge of “where” each sub-step should be applied (Roșca, 2009).

An impaired execution of complex arithmetic procedures has been reported also in basal ganglia and thalamic lesions (Corbett et al., 1988; Benke et al., 2003; Delazer et al., 2004; Roșca, 2009; Zamarian et al., 2009). The patients had associated deficits in fronto-executive functions (e.g. problem solving, cognitive set shifting, mental flexibility), and the calculation impairments were attributed to the disruption of the dorso-lateral prefrontal circuit involving the basal ganglia (Delazer et al., 2004). However, recently, the study of a patient with basal ganglia dysfunction revealed that his calculating deficits could not be exclusively linked to a sequencing disorder as suggested for previous single case reports; the analysis of his impairments spoke in favor of a pervasive arithmetic disorder (Zamarian et al., 2009).

Procedural knowledge has certainly been less investigated in basal ganglia lesions in comparison to other calculation abilities and, unfortunately, no analysis of the procedural impairments has been performed to date.

Taking a closer look at case studies reported so far, patients with impaired procedures described in the literature present visuo-spatial disturbances but preserved language abilities. For example, patients AS and PJ reported by Benke et al. (2003) had low scores in the Corsi Block span and Rey-Osterrieth Complex Figure tests. PJ also presented impaired performance in the VESPAR test (Verbal and Spatial Reasoning Test; Langdon and Warrington, 1995). HGM, a patient reported by Delazer et al. (2004) showed severe deficits in the Raven test; his scores in two subtests of the spatial reasoning test (VESPAR) were in the lower average (but very likely below his premorbid performance) and he had a slightly impaired

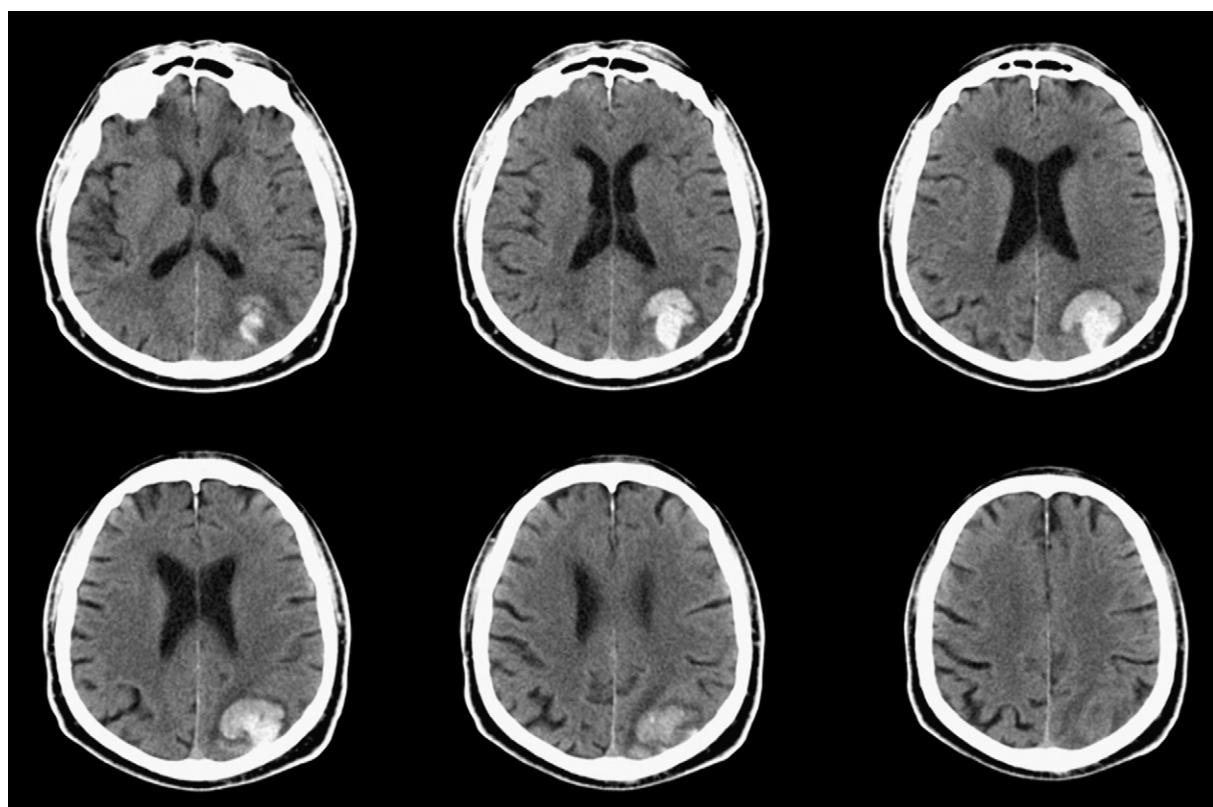


Fig. 1 – CT scan—patient SK.

estimation of space. Patient BD studied by Roșca (Roșca, 2009) scored on average for the Raven's Standard Progressive Matrices (49/60), although it is likely that her performance was below her premorbid status. Other cases with impaired procedural knowledge after parietal lesions have also been reported to have impaired visuo-spatial processing (e.g. Delazer et al., 2006).

The rationale behind the present study is to assess thoroughly the arithmetic impairment present in basal ganglia lesions, with a main focus on procedural knowledge, and to compare it with the procedural deficit resulting from left parietal lesions, which has been studied in more detail in the literature. Furthermore, we aim to disentangle whether these difficulties have a common basis such as impairment in the visuo-spatial working memory.

## 2. Results

### 2.1. Case description

SK was a 72-year-old, right-handed man with 18 years of formal education. The initial neurological examination revealed a mild motor deficit of the right upper limb, which resolved completely within a few days. The CT scan performed in the acute state showed a 3.6/3.2/3 cm left parieto-occipital hemorrhage, without mass effect (see Fig. 1).

CP was a 52-year-old, right-handed man with 13 years of formal education. The neurological assessment revealed a very mild right side hemiparesis which remitted completely

within 3 days. The CT scan performed at the hospital admittance showed a 1.5/1 cm hemorrhagic lesion located in the left lentiform nucleus and left internal capsule (see Fig. 2).

Both patients were pre-morbidly familiar with calculation. Neuropsychological testing (performed 1 month after the stroke for SK and 10 days after hospital admittance for CP) showed no aphasia (evaluated with Western Aphasia Battery, Kertesz, 1982), no apraxia (ideomotor, ideatory) and no right-left confusion. However, their writing was slow, with scrawled letters and their drawings to command were, although correct, somewhat simplified. On Raven's Standard Progressive Matrices, both patients had very low scores when compared to healthy subjects (SK—23/60 and CP—14/60). Their digit span was 6.

### 2.2. Number processing and calculation tasks

SK and CP were tested on a wide range of numerical abilities (see Table 1).

Both patients made no errors in counting by 1 (forward and backward). SK made some mistakes in counting by 7 from 1 to 50 with errors in the last 2 sums (28.5% errors), but counting backwards from 100 to 50 was severely impaired (85.7% errors). CP had no errors in counting by 7. In the dot counting task, SK was errorless and CP made 1 mistake (16.7% errors) in the item containing 12 dots.

In the transcoding task, SK made no errors when reading aloud Arabic numbers or number words; he correctly wrote number words and Arabic numbers to dictation, but made 2 mistakes in transcoding from Arabic code to orthographic

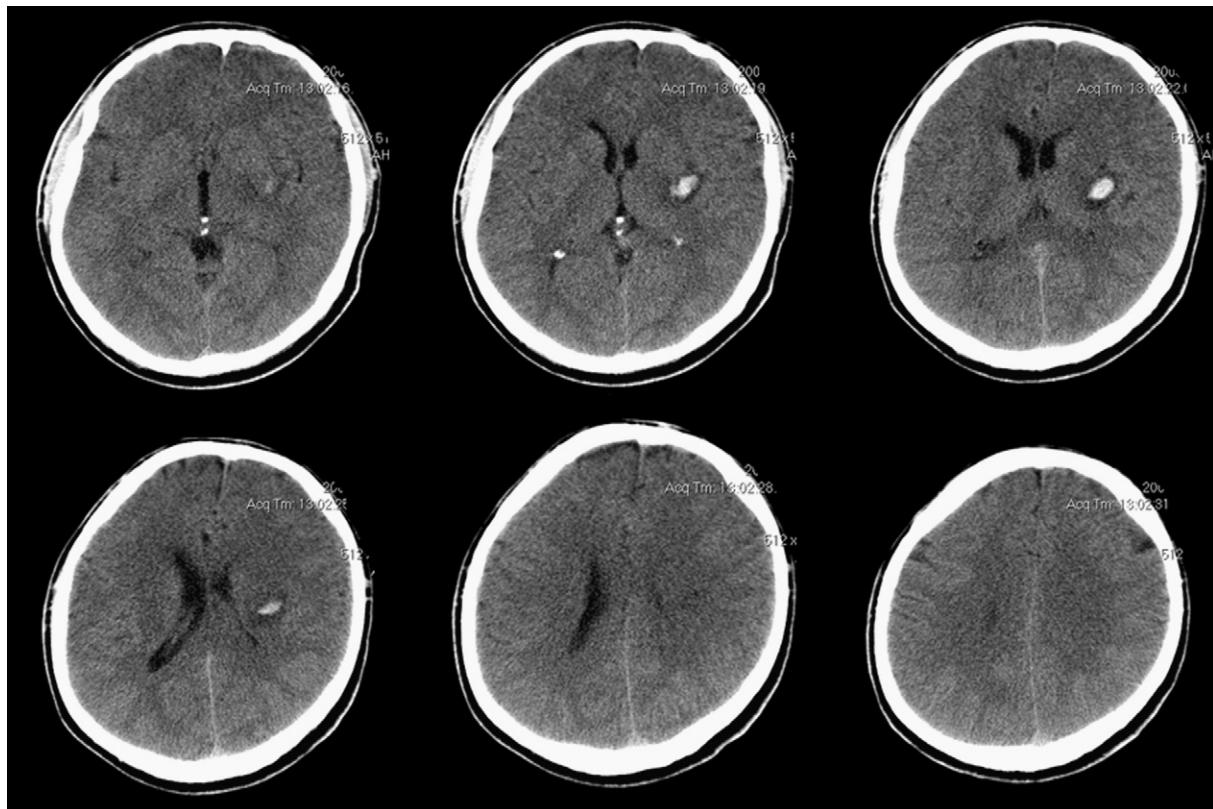


Fig. 2 – CT scan—patient CP.

**Table 1 – Number processing and calculation assessment.**

Task	Number of items	Errors (%)	
		Patient SK	Patient CP
Counting			
Counting by 1 from 1 to 20	20	0%	0%
Counting by 1 from 30 to 20	10	0%	0%
Counting by 7 from 1 to 50	7	28.5%	0%
Counting by 7 from 100 to 50	7	85.7%	0%
Dots counting	6	0%	16.7%
Transcoding			
Reading Arabic numbers	10	0%	10%
Reading number words	10	0%	0%
Writing number words to dictation	10	0%	10%
Writing Arabic numbers to dictation	10	0%	0%
Transcoding Arabic numerals into number words	10	20%	0%
Transcoding number words into Arabic numerals	10	20%	10%
Number comparison			
Comparing Arabic numbers	10	0%	0%
Comparing written number words	10	0%	10%
Simple arithmetic			
Addition	15	0%	0%
Multiplication	15	6.7%	26.7%
Subtraction	15	0%	0%
Division	15	40%	6.7%
Arithmetical rules			
Addition	4	25%	0%
Multiplication	6	16.7%	16.7%
Subtraction	6	16.7%	16.7%
Division	6	50%	66.7%
Mental arithmetic	20	70%	30%
Written complex arithmetic	16	75%	100%
Fractions			
Comparing fractions	4	25%	100%
Operations with fractions	4	50%	100%
Text problems	10	40%	40%
Arithmetical signs knowledge	10	0%	0%
Definition of operations	4	0%	0%
Approximation	12	25%	33.3%
Cardinality estimation			
Positioning Arabic numbers on a scale	10	0%	0%
Positioning word numbers on a scale	10	0%	0%
Contextual magnitude judgment	5	0%	0%
Time estimation	5	0%	0%
General numerical knowledge	5	0%	0%
Personal numerical knowledge	4	0%	0%
Digit span	5	0%	0%

code (20% errors); also, he made 2 errors in transcoding from written number words to Arabic code (20% errors) with 1 inversion and 1 omission. CP provided correct answers in reading aloud number words but made 1 omission when reading Arabic numbers (10% errors); he was errorless writing Arabic numbers by dictation, but made one mistake in writing number words by dictation due to his agraphia (10% errors); he transcoded correctly the Arabic numbers into orthographic form, but made one omission in transcoding the written number words into Arabic numerals (10% errors).

Both patients answered correctly the simple additions and subtractions. In the multiplication tables SK made one mistake (6.7% errors), saying that  $4 \times 5 = 25$  and CP provided 4 incorrect responses (26.7% errors), making 2 table value errors (e.g.  $7 \times 5 = 40$ ) and 2 non-table value errors (e.g.  $4 \times 9 = 56$ ;  $7 \times 9 = 61$ ). The division task showed more impairment in SK

(40% errors) than in CP (6.7% errors). SK tried many times to compute the divisions by relying on multiplications; for example, when asked for the result of  $20: 5$  he said “ $5 \times 4 = 20$ , so the answer is 4.”

SK made some mistakes in rule-based problems (e.g.  $n+0$ ,  $n \times 0$ ): one in an addition ( $7+0 = 0$ ), one in a multiplication ( $0 \times 5 = 5$ ), one in a subtraction ( $6-0=0$ ) and three in divisions (always saying that  $n: n = n$ ). CP also made mistakes, one in a multiplication ( $4 \times 0 = 4$ ), one in a subtraction ( $3-0=0$ ) and in 4 divisions (always responding that  $n: 1=1$  and once that  $n: n=0$ ), with 18.2% errors.

In the mental calculation task, SK made many errors (70% errors). He was only able to mentally compute the easiest operations ( $2d+1d$ ,  $2d-1d$ ,  $2d \times 1d$ ,  $2d: 1d$ ) those that did not necessitate carrying or borrowing steps. When allowed to use paper and pencil, he did not apply correctly the carrying and

the borrowing rules, claiming that he did not remember them anymore. For example, for  $31+87$  he computed that  $1+7=8$ , he wrote 8,  $8+5=13$ , the final result being 138; for  $55-28$  he subtracted 8 from 15 saying he borrowed 1, but in the next step he borrowed 1 from five and added 2 to 4 instead of subtracting 2 from 4, the final result being 67. In the multiplication tests, he said he did not remember which number was supposed to be written down and which was not; when asked to resolve  $24\times 7$ , he correctly computed that  $7\times 4=28$ , wrote 8 in the last column but did not know what to do with 2, and was unable to finish the operation. When trying to compute  $85:5$ , he first wrote 1 but did not know what to do with the 3 that he obtained from subtracting 5 from 8. He tried to use back-up strategies without success, always stopping when the problem implied carrying or borrowing.

CP managed better than SK in the mental calculation task (30% errors), but substituted usual procedures for back-up strategies. For example, when calculating  $31+87$ , he decomposed the numbers as follows:  $30+80=110$ ,  $110+1=111$  and  $111+7=118$ . For more demanding calculations, such as  $17\times 8$  he was unable to keep track of the operations: he started to compute  $10\times 8=80$ ,  $7\times 7=49$  but could not continue claiming that there were too many steps. His incorrect responses were very close to the correct answer (e.g. 45 instead of 43; 23 instead of 27). When asked to use his procedural knowledge to solve the items instead of using back-up strategies, CP stated that he could not remember the rules anymore; the patient said that allowing him to use paper and pencil would not help him as he did not remember how to compute such calculations. In both patients, it was impossible to find any consistent “bugs.”

As occurred in the precedent task, when confronted with the written complex calculation task, SK was able to answer correctly only those items that did not necessitate borrowing and carrying steps (75% errors). He made some interesting mistakes: in subtractions, he added digits instead of subtracting them (e.g. when working on  $473-84$ , he added  $3+4$ , wrote 7, then added  $7+8$ , wrote 5, and in the last step subtracted 1 from 4, the result being 357). In multiplications, he correctly answered that  $34\times 9=306$ , but instead of using carrying rules he wrote:

$$\begin{array}{r} 34 \times \\ 9 \\ \hline 36 \\ 27 \\ \hline 306 \end{array}$$

In complex additions and subtractions he always asked “How much do I carry /borrow?” “Where do I carry/borrow from?” “Do I have to carry/borrow something?” The same carrying problem affected the multiplications. In divisions, he changed the order of the steps (e.g.  $456:3=133$ ; he said that 3 in 4 goes once, wrote 1, subtracted 3 from 4, obtained 1 and wrote it under 4, then divided 6 into 3 obtaining 3, and for the last step he divided 15 into 3 and wrote another 3).

$$\begin{array}{r} 456 : 3 = 133 \\ 15 \end{array}$$

When asked to compute complex written calculations, CP said he could not perform them because he did not know the

rules and the numbers were too large to decompose in smaller numbers in order to perform the operations in the same manner as he had for the mental calculation tasks. He stated that he did not remember how it should be done and he did not provide answers (100% errors).

When asked to compare fractions, SK made one mistake when he stated that  $3/15$  was larger than  $5/15$  (25% errors). For calculations using fractions, he provided incorrect responses for 50% of the items. He was only able to compute the easiest operations ( $1/2+1/2$  and  $3/5-1/5$ ) and said that he did not remember the rules anymore. CP provided no answer for these tasks and stated that he had “forgotten” the meaning of fractions (100% errors).

For text problems, the patients were only able to resolve correctly the problems that required simple operations, producing 40% errors.

Both patients were able to recognize and name the arithmetical signs and also, when dictated, wrote them correctly. They provided adequate answers when asked to define the four arithmetical operations.

For the number comparison task, SK was errorless and CP made 1 error when he indicated that “two thousand five hundred and three” was larger than “five hundreds and three” (10% errors).

For the approximation task, both patients made a few mistakes with a rate of error of 25% in SK and 33.3% in CP. However, for other tasks assessing quantitative and non-quantitative semantic number knowledge (cardinality estimation, contextual magnitude judgment, time estimation and general and personal numerical knowledge) both patients were accurate.

### 3. Discussion

#### 3.1. Acalculia patterns

By analyzing the pattern of acalculia of patients SK and CP, it becomes clear that both have impaired procedural knowledge (see Table 2). Their deficit cannot be explained by impairment in the understanding of the arithmetical signs or concepts

**Table 2 – Items that require the use of arithmetic procedural knowledge.**

Task	Number of errors	
	Patient SK	Patient CP
Complex mental arithmetic		
Addition	4/5	1/5
Subtraction	4/5	1/5
Multiplication	3/5	1/5
Division	3/5	3/5
Complex written calculation		
Addition	3/4	4/4
Subtraction	3/4	4/4
Multiplication	3/4	4/4
Division	3/4	4/4
Operations with fractions		
	2/4	4/4
Text problems		
	4/10	4/10

behind the procedures. They are able to recognize correctly the arithmetic signs and to define the operations. In addition, we can exclude failure on complex calculation due to transcoding problems because the number of transcoding errors was significantly reduced when compared to their inability to compute complex calculations. Although the patients made mistakes in tasks involving arithmetic facts and rules, these do not seem enough to explain all of their mistakes in complex calculations.

It was not possible to find any consistent “bugs” by analyzing the errors made by the patients in mental and written calculation. Their deficit could be of “memory” type since both stated that they did not remember the procedures and did not know how, when and what to compute. Furthermore, they did not show a decrease in performance as the operation proceeded and they were aware of their deficit.

Patient SK, after a left parietal lesion, presented errors in complex mental and written calculation due to an impaired knowledge of how, what and when to compute. His errors and the questions that he formulated to the examiner (e.g. which number should be written down, where to put the numbers that resulted after a carrying operation, where he should borrow from) could reflect a spatial deficit, making the patient unable to adequately control the spatial attention and working memory necessary for multidigit calculations.

Patient CP, after a left basal ganglia lesion, was also unable to compute complex mental and written operations, although he used back-up strategies to overcome his deficit. He decomposed the operands into smaller numbers and was successful as not many steps were needed. These findings suggest that, besides impairment in arithmetic procedural knowledge (clearly indicated by the patient’s verbal report) he also presented monitoring deficits, probably due to the disruption of the fronto-subcortical executive circuits.

In both patients, the impairment in procedural knowledge was also revealed by the tasks that required resolving operations with fractions and text-problems that necessitated complex calculations.

Prior reports of basal ganglia lesions have only mentioned the existence of these impairments, but no analysis has been performed. The present study suggests that, besides the executive dysfunction due to the disruption of the fronto-subcortical circuit, basal ganglia lesions produce a deficit in arithmetic procedural knowledge, which is very similar to the deficits that have been previously reported in left parietal lesions.

In addition to the impairments in arithmetic procedural knowledge, both patients made a few mistakes in simple arithmetic and arithmetic rule – based problems (e.g.  $n \times 0$ ,  $n : 1$ ). SK made errors in simple divisions and rule-based problems. Although some individual differences may be expected, simple subtractions are hypothesized to be computed through processing of analogue magnitude and require semantic elaboration (Dehaene and Cohen, 1995; Dehaene and Cohen, 1997; Dehaene et al., 2003). While multiplication tables and simple additions are assumed to be stored in long-term memory, data on simple divisions has been seldom reported. Therefore, it is an open question if they are represented as a subset in semantic memory or if

divisions and multiplications share the same connected fact representations. The dissociation between multiplications and divisions found in patient SK indicates possible separate storage, although the divisions can also be solved based on multiplication tables. Arithmetical rules have been reported to dissociate from other operations and can be selectively affected by injury (McCloskey et al., 1991; Delazer et al., 2004). In addition to impaired procedures, patient CP presented a few errors in multiplication facts and arithmetical rule-based problems. The deficit of fact retrieval in basal ganglia lesions has been described previously (Hittmair-Delazer et al., 1994; Dehaene and Cohen, 1997; Delazer et al., 2004; Zamarian et al., 2009), as well as the impairment in arithmetical rules (Delazer et al., 2004).

### 3.2. Anatomical substrate of procedural knowledge

From an anatomical point of view, the neural substrate for mathematical computing remains a highly debated topic. Even the triple code neuropsychological model (Dehaene, 1992; Dehaene and Cohen, 1995; Dehaene and Cohen, 1997; Dehaene et al., 2003) which is the only model that tries to relate the specific cognitive processes involved in arithmetical knowledge to anatomical substrates does not provide enough information on procedural knowledge and the brain areas involved in storing the algorithms that are necessary to solve complex operations.

Is it possible that the arithmetic procedural knowledge relies on a fronto-parieto-subcortical circuit that is not devoted exclusively to complex calculation but also to a general visuo-spatial memory process? Recent imaging studies have shown that a wide range of visuo-spatial memory and attentional areas are engaged in complex calculation tasks (Zago et al., 2001; Pesenti et al., 2001; Zago and Tzourio-Mazoyer, 2002), including the widespread activation of a fronto-parietal network (Grabner et al., 2009) and the recruitment of basal ganglia when the calculation task is complex or the presentation rate is high (Menon et al., 2000; Kong et al., 2005).

The basal ganglia receive input from many regions of the cerebral cortex, including the prefrontal and posterior parietal cortical areas (e.g. Yeterian and Pandya, 1993; Strick, 2004), with a pallidal output to the same cortical regions via thalamic nuclei (Kalivas et al., 2001; Clower et al., 2002). They also receive projections from the thalamus, where feedback circuitry exists between these brain regions (for a review see Mengual et al., 1999). Thus, a major structural feature of the basal ganglia is their participation in at least five separate loops with the cerebral cortex (Alexander et al., 1986) and the potential to influence the control of higher cognitive functions. These circuits may constitute the anatomical basis for arithmetic procedural knowledge and for visuo-spatial processing.

### 3.3. Neuropsychological substrate of procedural knowledge

A possible explanation for the impaired procedures in patients SK and CP could be that the arithmetic procedural knowledge relies on a well-established routine based on an overlearned

spatial layout that specifies how, what, when and where each sub-step of the procedure must be carried out. The fact that both patients had a low score on Raven's Standard Progressive Matrices, that their drawings were somewhat simplified and that there was no aphasia suggests a subjacent visuo-spatial disturbance.

The visuo-spatial working memory implying the active system for temporarily storing and manipulating the information needed for the execution of complex cognitive tasks (Baddley, 1986) has been suggested to be involved in the computing of complex arithmetical operations (Hayes, 1973). However, some authors have claimed that performing complex calculations necessitates verbal working memory (Hitch, 1987). In the present case studies, both patients showed an intact verbal memory at the digit span test. Their visuo-spatial working memory deficit was the most possible cause for their calculation impairment. Furthermore, both SK and CP had low scores on Raven's Standard Progressive Matrices tasks that necessitate of visuo-spatial working memory.

Is a fronto-parieto-subcortical circuit a suitable anatomical basis for visuo-spatial working memory? Studies reported in the literature indicate that the visuo-spatial working memory depends on interactions between the prefrontal cortex and the posterior parietal neocortex (Chafee and Goldman-Rakic, 2000; Miller and Cohen, 2001), of which the prefrontal cortex controls the maintenance of spatial codes through reciprocal interactions with the posterior representations. In addition, the basal ganglia have been demonstrated to play an important role in these cognitive processes by functional neuroimaging (Menon et al., 2000; Lewis et al., 2004) and inactivation studies (Kalivas et al., 2001). Their involvement in visuo-spatial cognition has been suggested on the basis of their connection to the two major streams of the visual system (for a review see Packard and Knowlton, 2002).

#### 3.4. Conclusions

In conclusion, the analysis of the cases presented in this work, in conjunction with other cases in the literature and anatomical and functional studies points to the existence of a possible visuo-spatial schema that supports the demands of complex calculation. The anatomical substrate for arithmetical procedural knowledge might be a left fronto-parieto-subcortical loop, in which the fronto-parietal and fronto-subcortical loops provide support for the monitoring and sequencing of the necessary steps of a complex calculation, and the parieto-subcortical loop supports the visuospatial working memory. Patient SK, due to a left parietal lesion presented impaired arithmetic procedural knowledge with a deficit in visuo-spatial working memory as the most probable substrate. In the case of patient CP, the damage of basal ganglia interrupted not only the fronto-subcortical loop (that mediates the correct sequencing of single steps within the solution and the monitoring of the solution), but also the parieto-subcortical loop responsible for a more pervasive disorder that implies also procedural knowledge. Accordingly, he had some problems with monitoring that were evident when he was not able to use back-up strategies successfully when calculations needed numerous steps (e.g. complex written calculation). He also presented a genuine arithmetic procedural knowledge

deficit, not remembering the rules necessary to compute complex calculation.

Limitations of this study include the fact that the patients were tested in the acute stage of the stroke and there are reports showing that many cognitive deficits are present only in the acute phase associated with a significant deficit of cortical vascular perfusion. The lesions in SK and CP were evidenced using CT-scan, which is not a good tool for excluding diaschisis. However, the rationale for testing the patients early after the stroke was that after injury lost functions are recovered due to neuroplasticity, resulting in the improvement of some deficits. Furthermore, the patterns of acalculia previously reported in the literature are similar to those found in our patients, although they are due to degenerative diseases or old vascular lesions, leading us to assume that the present findings are reliable. Additionally, it may be argued that SK and CP were not tested thoroughly in order to gain more information on their visuo-spatial abilities; unfortunately that was not possible. However, the present work can be a starting point for further investigations in this direction, which can emphasize on visuo-spatial working memory.

In summary, the present paper analyzes and compares for the first time the deficits in calculation and number processing found in two distinct cerebral lesions (left parietal and left subcortical). The data obtained, which is corroborated by other studies, indicate a possible left fronto-parieto-subcortical circuit responsible for arithmetic procedural knowledge that relies on a visuo-spatial sketchpad containing a layout representation for each sub-step of the procedures.

#### 4. Experimental procedures

Both subjects gave their informed consent to participate in the study.

Number processing and calculation tasks:

The patients were tested on a broad range of numerical abilities; the same items were administered to both (see Table 1).

They had to count forward by 1 from 1 to 20 and by 7 from 1 to 50 and backward by 1 from 30 to 20 and by 7 from 100 to 50. Additionally, there were 6 dot counting tasks, up to 12 black dots being presented on a sheet of paper in a regular pattern (as the numbers in dice) and in random patterns. Dots were always of the same size, there was no time limit and the answer was provided orally.

In the transcoding task, 6 subtests were administered, including all possible transcoding routes among Arabic numerals, spoken and written number words (10 items for each possible route). The items of each subtest were constructed with reference to the same syntactical structure, their length varying from one to five digits.

The performance in simple arithmetic was tested by presenting the patients with 60 problems in Arabic format. They had to answer to 15 one-digit additions, 15 one-digit multiplications, and 15 one-digit subtractions with numbers ranging from 1 to 9. For the division task, there were 15 items comprising division facts (e.g. 6:3, 14:7). The patients also had to answer 4 rule-based additions ( $n+0$ ), 6 rule-based multiplications ( $n \times 0, n \times 1$ ), 6 rule-based subtractions ( $n-n, n-0$ )

and 6 rule-based divisions ( $n: 1$ ,  $n: n$ ), as these operations have been demonstrated to be stored separately in memory (e.g. McCloskey et al., 1991).

For the mental calculation task, SK and CP had to mentally compute 20 problems presented visually in Arabic format, each one on a separate sheet of paper. The following structures were included (d=digit):

Addition:  $2d+1d=2d$  with carrying (1 item),  $2d+2d=2d$  without carrying (1 item),  $2d+2d=2d$  with carrying (1 item),  $2d+2d=3d$  with carrying (2 items).

Subtraction:  $2d-1d=2d$  without borrowing (1 item),  $2d-2d=1d$  with borrowing (1 item),  $2d-2d=2d$  without borrowing (1 item),  $2d-2d=2d$  with borrowing (2 items).

Multiplication:  $2d \times 1d=2d$  without carrying (2 items),  $2d \times 1d=3d$  with carrying (3 items).

Division:  $2d: 1d=2d$  (3 items),  $2d: 2d=1d$  (1 item),  $3d: 2d=2d$  (1 item).

In order to assess the patients' ability to resolve complex arithmetic problems, they were asked to compute 16 complex written calculations that were presented in Arabic format in a straight horizontal alignment, with every item on a separate sheet of paper. The structure was as follows:

Addition:  $3d+3d=3d$  (1 item),  $3d+2d=3d$  (1 item),  $3d+3d=4d$  (1 item),  $4d+3d=4d$  (1 item); all items except the first one implied carrying.

Subtraction:  $3d-3d=3d$  without borrowing (1 item),  $3d-2d=3d$  with borrowing (1 item),  $3d-3d=3d$  with borrowing (1 item),  $4d-3d=4d$  with borrowing (1 item).

Multiplication:  $2d \times 1d=3d$  (1 item),  $3d \times 1d=4d$  (1 item),  $3d \times 2d=4d$  (2 items); all the operations necessitated carrying.

Division:  $3d: 1d=3d$  (1 item),  $3d: 1d=2d$  (1 item),  $3d: 2d=2d$  (2 items).

Both patients were asked to verbalize their thoughts during complex mental and written calculation tasks.

The use of fractions was tested by asking SK and CP to compare 2 fractions (4 items) and indicate the larger value and to resolve additions and subtractions with fractions (4 items). In order to evaluate the patient's ability to solve 10 numerical problems that are embedded in real life situations, they were asked to respond verbally to text problems (e.g. If it is 9:30 now and you have an appointment at 11:35, how much time is left?), being allowed to use paper and pencil if they considered it necessary.

For the arithmetical signs knowledge task, the patients were asked to name the arithmetical signs presented by the examiner (each one on a white sheet of paper) and to write them after dictation. Additionally, they were asked to define the four arithmetical operations.

For the number comparison task, the patients were asked to indicate the larger number from 20 pairs of numbers up to 5 digits, 10 being presented in Arabic format and 10 in orthographic form; each pair was presented separately on a white sheet of paper.

The patient's ability to approximate the result of an operation was tested by asking them to "estimate" as quickly as possible the result of 12 complex arithmetic operations (3 additions, 3 subtractions, 3 multiplications and 3 divisions); they had to choose the answer that they considered to be the closest to the correct result from a group of 4 false results. For a cardinality estimation task, SK and CP were required to

indicate the position of a given Arabic numeral (10 items) and a spoken number word (10 items) on an analog vertical number scale that extended from 0 at the bottom to 100 at the top. In a contextual magnitude judgment task, the patients had to indicate whether a number of objects represented a small, medium or large quantity in a given context (e.g. a 15 pages novel, a 20 kg elephant). Their time estimation ability was tested by asking them to estimate the elapsed time since a well-known event took place (e.g. how long ago was the current president elected?).

Other questions were used to test their general numerical knowledge—5 items (e.g. how many months are in 1 year?) and their personal numerical knowledge (their address, phone number and date of birth).

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